# A New Ocean Wave Model Applied to Humboldt Bay Entrance

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## Abstract

- Humboldt Bay has a dangerous entrance channel due to steep waves.
- This poster describes our first attempt to model and simulate these waves only accounting for horizontal water motion along a mid-channel axis, the x-axis.
- In the future we plan to include vertical water motion and forcing other than bottom friction.


## Methods: Continuity Equation

- As water enters the bay, it is constrained by rock jetties to the North and South and by the bottom. It is also squeezed between incoming ocean swell and outgoing bay water. Without anywhere else to go, it rises into a wave.
- Mathematicians describe this effect with the continuity equation. If $u$ is the water velocity and $h$ is the water height, then

$$u\frac{\partial h}{\partial x} + h\frac{\partial u}{\partial x} = -\frac{\partial h}{\partial t} \qquad (1)$$

- This says that water slopes $\partial h/\partial x$ and water velocity gradients $\partial u/\partial x$ cause waves to rise or fall at speeds of -$\partial h/\partial t$ .

## Methods: Momentum Equation

- The motion of water is also governed by Newton's Second Law, force equals mass times acceleration. F=ma.
- Since the water's velocity depends on both space and time $t$, acceleration is computed with a mathematical rule known as the chain rule.

$$a = \frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$$

- The force per unit mass on a parcel of water is produced by gravitational restoring forces and bottom friction, so Newton's Law implies

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -g\frac{\partial h}{\partial x} - \lambda u\,|u| \qquad (2)$$

- This momentum eq. must be digitized and solved simultaneously with eq. 1.

## Wave Theory

- The bay entrance is shallow compared to the wavelength of ocean swell. Theory implies that all waves entering the bay must travel at the same speed $\sqrt{gD}$ meters/second, where g is the acceleration of gravity and D is the depth. In model units this speed is 1.0.

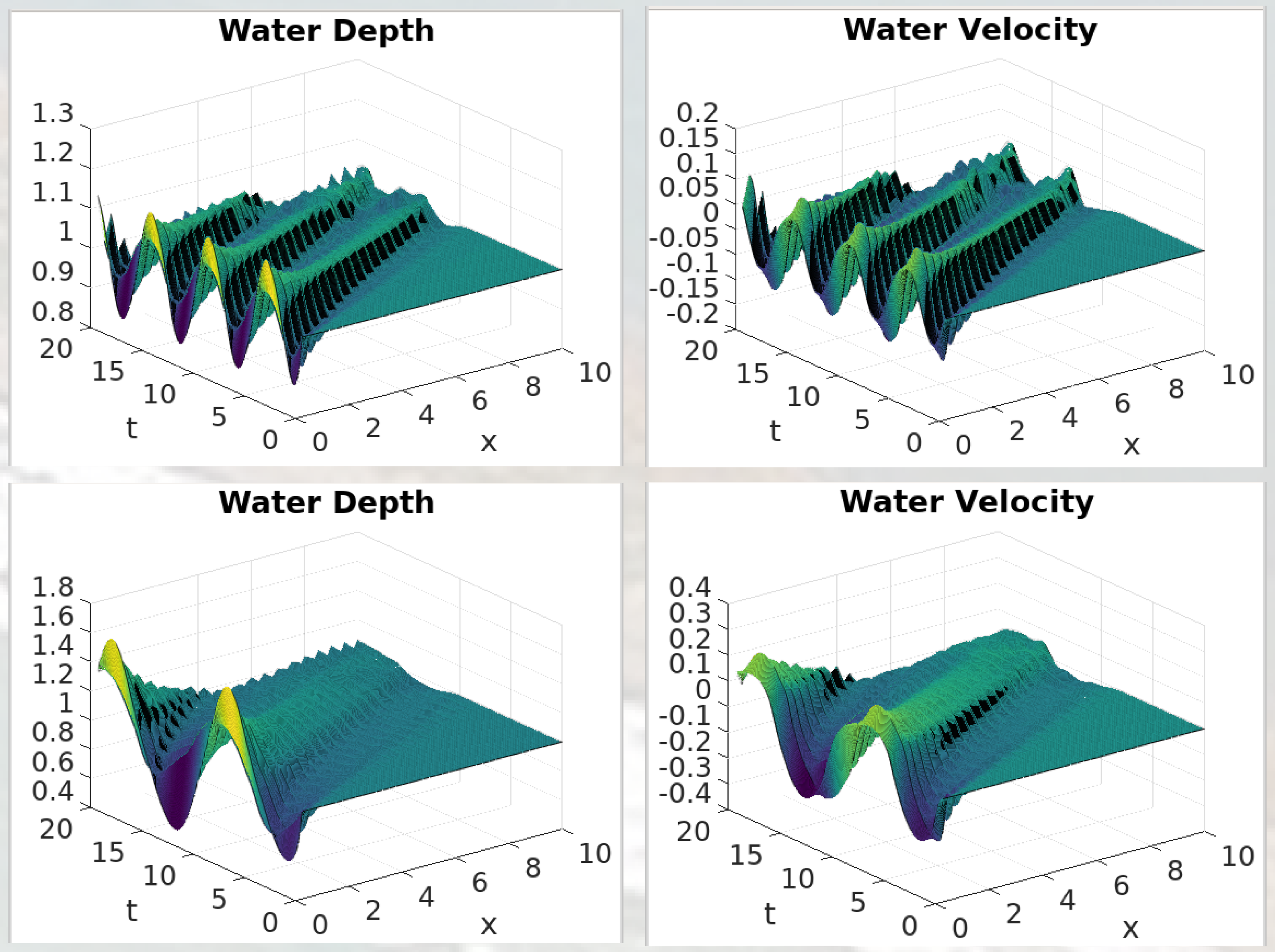
## Digitized Continuity Equation

- When digitized on a grid for the computer, equation (1) becomes

$$-\frac{u_{i-1}^{j+1}}{4\beta}h_{i-1}^{j+1} + h_i^{j+1} + \frac{u_{i+1}^{j+1}}{4\beta}h_{i+1}^{j+1} = \frac{u_{i-1}^{j}}{4\beta}h_{i-1}^{j} + h_i^{j} - \frac{u_{i+1}^{j}}{4\beta}h_{i+1}^{j}$$

- Time superscripts and space subscripts give grid locations. $\beta = \frac{dx}{dt}$.
- A similar result is obtained for eq. (2).

## Results




## Conclusion

Predicted model wave speeds equal the propagation slopes in the *x-t* planes above. Within estimation error, model wave speeds for five swell scenarios agree with theory.