Improved Measurement of the Newtonian Gravitational Constant *G*

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Introduction

Historical Significance

The Newtonian gravitational constant, G, was one of the first fundamental constants discovered by humans, yet it remains the least precisely known. Specifically, G arises in Newton's law of universal gravitation (1), first published in 1678. The first indirect measurement of G was made by Henry Cavendish in 1797-98, which resulted in the value $G = (6.67 \pm 0.07) \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$.

$$F = G\frac{Mm}{r^2} \quad (1)$$

Motivation

Measuring G is exceedingly difficult. Since G is not related to other fundamental constants by any complete theory, it's value can only be determined experimentally. Yet, the precision of G has improved by only roughly two orders of magnitude, and modern measurements of G continue to disagree. The precision measurement of G has implications in gravitational physics, as well as cosmology and general metrology.

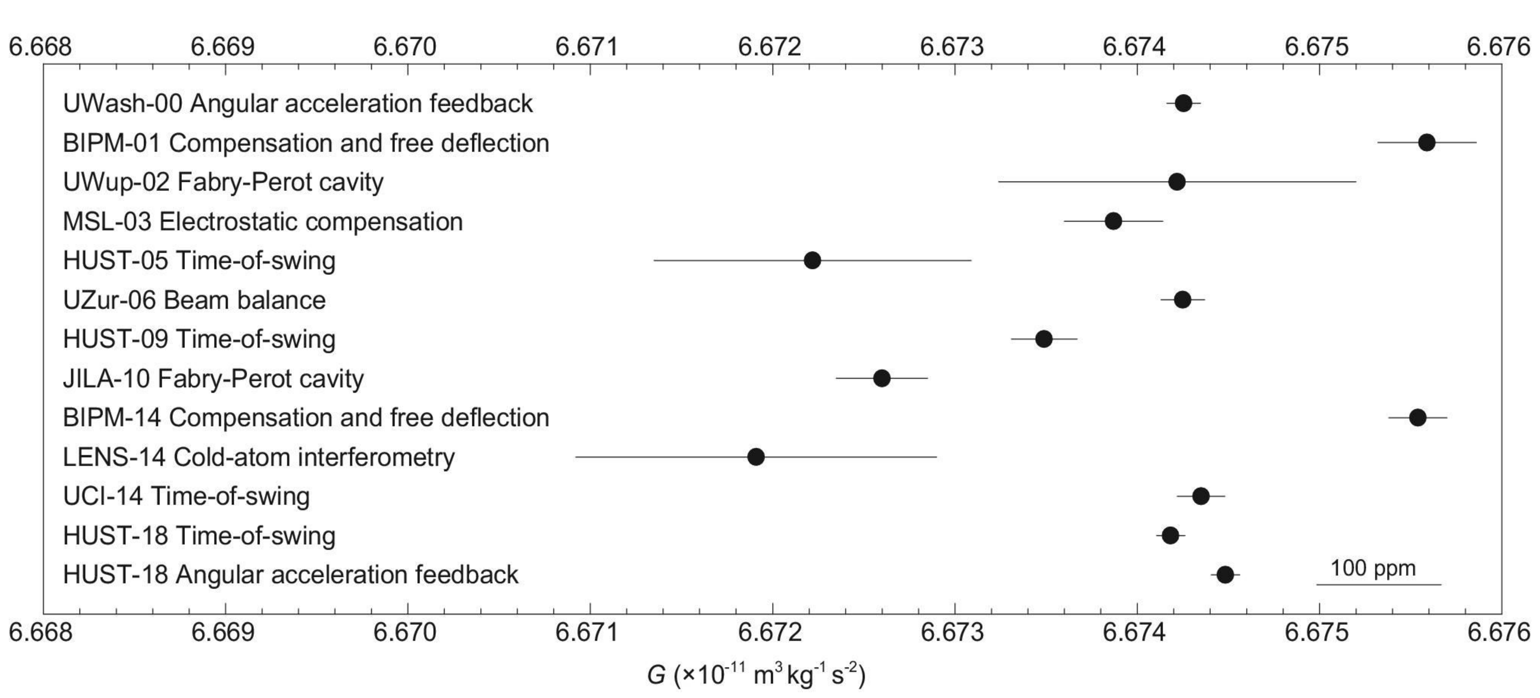
Table 1 - CODATA values of G and relative error from the past 50 years [1]. Additionally, the most recent CODATA value from 2018 has been updated to $G = 6.674\ 30\ (15) \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$ [2].

CODATA	$G\ (\times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}})$	u_r (ppm)
CODATA 1973 [14]	6.6720 ± 0.0041	615
CODATA 1986 [15]	$6.672\,59 \pm 0.000\,85$	128
CODATA 1998 [16]	6.673 ± 0.010	1500
CODATA 2002 [17]	6.6742 ± 0.0010	150
CODATA 2006 [18]	$6.674\,28 \pm 0.000\,67$	100
CODATA 2010 [5]	$6.673\,84 \pm 0.000\,80$	120
CODATA 2014 [4]	$6.674\,08 \pm 0.000\,31$	47

Previous Measurement

Over 200 measurements of G have been made, using several different methods. The results vary considerably, and modern measurements continue to reveal values of G that do not agree within their relative error. Table 2 shows the spread of 13 such measurements. Note that even precise measurements (with relative uncertainties of 12 ppm) performed by the HUST group in 2018 differ significantly between the time-of-swing method and the angular acceleration feedback method.

Table 2 - Measurements of G and relative errors that have been made since 2000, with group, year, and method noted on the left [1].



Experimental Design

Methods and Apparatus

This experiment is modeled after the methods and apparatus used by the UWash group [3], and similar to the HUST group [1]. In 2000, Gundlach and Merkowitz at the University of Washington pioneered the angular acceleration feedback technique, resulting in a measurement of G with an accuracy of 14 ppm [3]. In 2018, the HUST group ran parallel tests using angular acceleration feedback and time-of-swing, resulting in two different values of G, both with an accuracy of 12 ppm [1].

- *Angular acceleration feedback* - works by first rotating the outer turntable at a constant rate, then turning on feedback to minimize the twist in a torsion fiber, resulting in an angular acceleration of the inner turntable that is equal to the gravitational acceleration.
- *Time-of-swing* - utilizes the frequency of oscillation of the pendulum for different arrangements of the test masses, and derives the gravitational signal from the period of these oscillations.

This project is the first to utilize these two methods of measuring G within the same apparatus, in order to disentangle systematic effects related to the technique of measurement used.

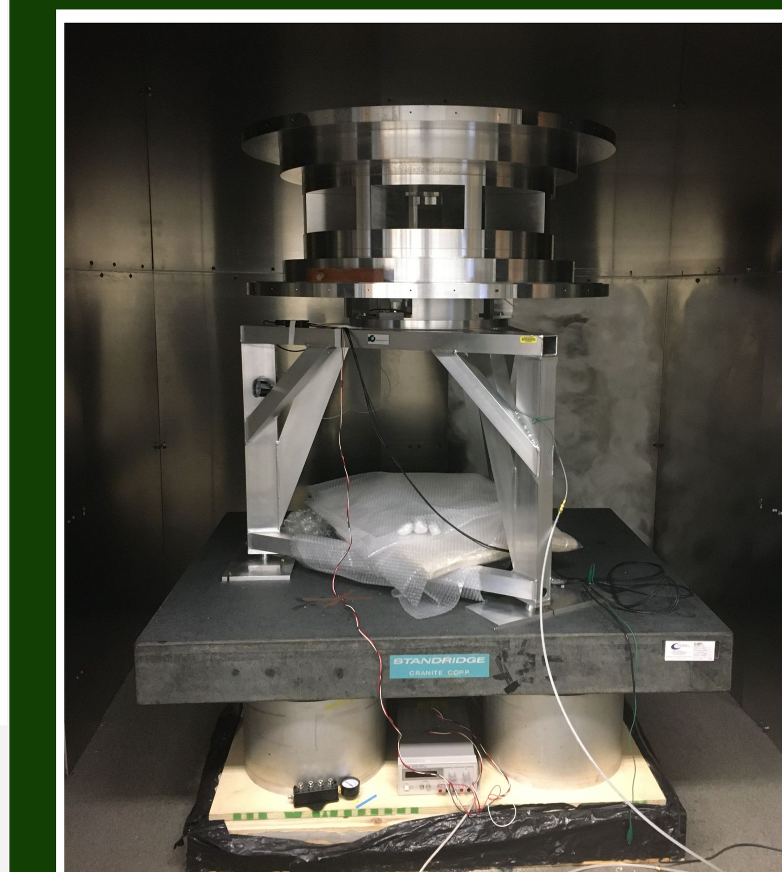


Figure 1 - The assembled outer turntable of the physical apparatus at IUPUI.

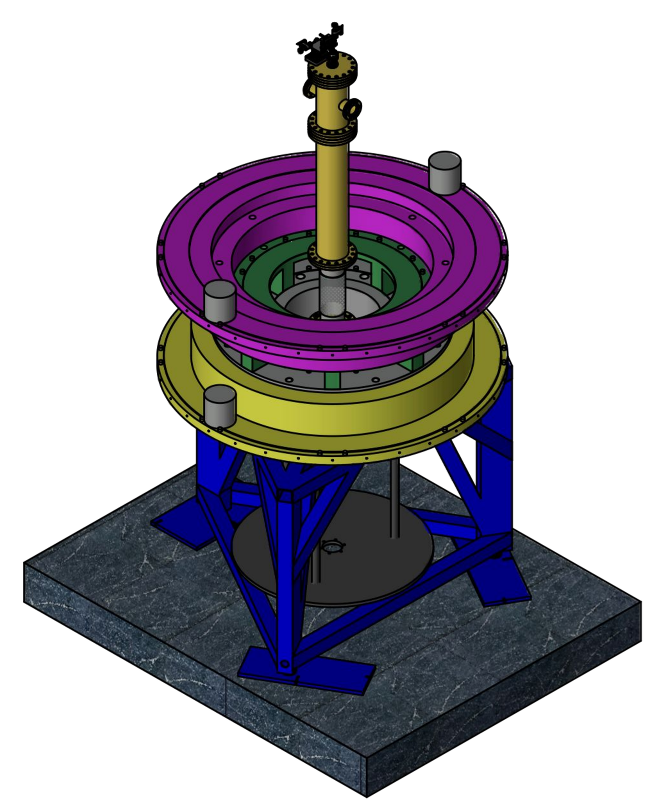


Figure 2 - A schematic of the full apparatus including the outer turntable and test masses. Note that the coloring has no physical meaning.

Minimizing Uncertainty

This apparatus is uniquely designed for the smallest relative uncertainty for any measurement of G to date. Some of these design components are:

- Using cylindrical test masses cut from a single silicon crystal to decrease uncertainty related to density inhomogeneity.
- Scale size is almost twice the size of similar apparatuses, which reduces the relative error in characterization measurements.
- Interferometric position sensing to allow for real time tracking of attractor masses.
- Utilizing the same apparatus for both methods allows for easy interchange of technique without changing the experimental set up.

It is important to reiterate that measuring G using both methods within the same apparatus allows analysis of deviations in the experimental value of G due to the method of measurement.

Altogether, this experiment has an accuracy goal of 2 ppm for the relative uncertainty in G.

Current Status

Modeling the Expected Signal

To ensure the accuracy of the data to be collected, it is critical that the gravitational effects from the test masses be isolated from any noise in the experiment. Comprehensive calculations are being made using multipole analysis (2) of the expected angular acceleration feedback from possible test mass configurations on the turntable as well as the turntable itself [3]. Here, ϕ describes the rotation angle between the pendulum and the attractor mass, q_{lm} are the multipole moments of the pendulum, and Q_{lm} are the multipole fields of the turntable.

$$\alpha(\phi) = \sum_{l,m} \alpha_{l,m} = -\frac{4\pi G}{I} \sum_{l=2}^{\infty} \frac{1}{2l+1} \sum_{m=-l}^{+l} m q_{lm} Q_{lm} e^{im\phi} \quad (2)$$

Preliminary Measurements

Preliminary measurements are being made with a prototype pendulum so that its behavior can be understood. Initial measurements show that the pendulum has a large angular drift that will need to be addressed. Figure 3 shows that heat treatment of the fiber may help eliminate fiber drift issues of the pendulum for a clearer signal.

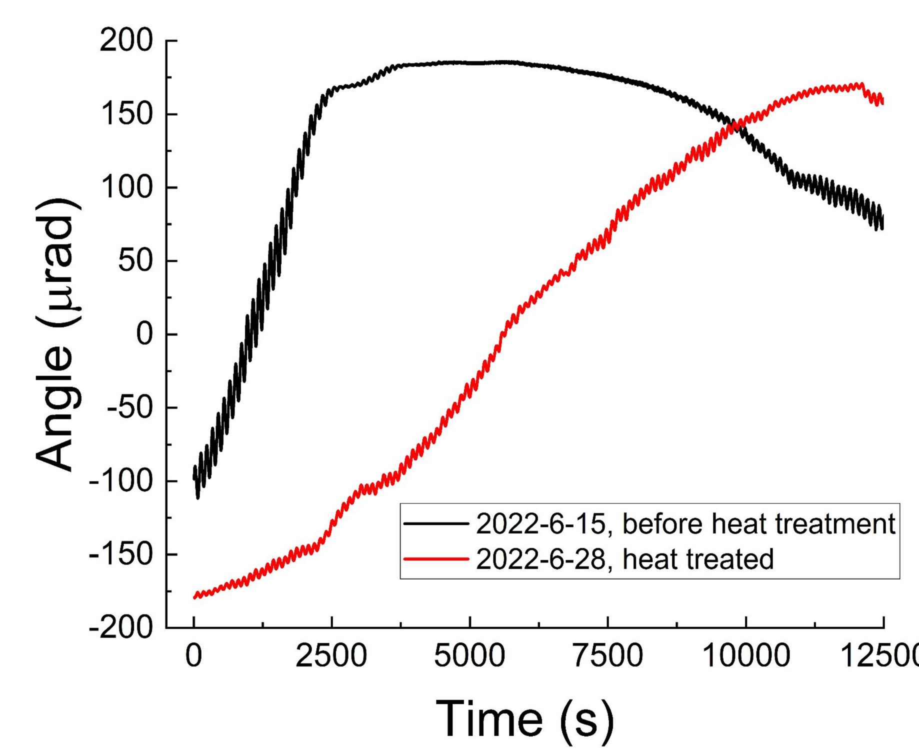


Figure 3 - Plot of the twist angle of the pendulum over about 3 hours. The black line shows the fiber drift before heat treatment, while red shows after. Ideally, this plot would show oscillations at the resonant frequency about a flat trend line.

References

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